Dynamical Model and Simulations for Gamification of Learning

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Abstract

In this paper an example for the dynamical model for gamification is considered and simulated. Based on the theories of Game Design Features (GDF - cool features, fancy graphics, challenging puzzles, and an intriguing setting and story), Key Characteristics of a Learning Game (KCLG - Challenge, Curiosity, Fantasy and Control), a theory of educational environment design model (ARCS - attention, relevance, confidence and satisfaction), and the theoretical background of Gamification labeled as the MDA (Mechanics, dynamics, and aesthetics) framework, four primary factors such as curiosity, challenge, fantasy and control have been originated. By using these four primary factors, the dynamical model for Gamification was developed and simulated. Furthermore the sensitivities of the four primary factors are considered for different individuals and simulations are represented. The examples and simulations make the values and characteristics of the four primary factors meaningful.

Keywords*: Gamification, Game Based Learning, DGBL, Learning Game, Serious game, Dynamical Model*

1. Introduction

The purpose of this paper is to simulate 'Dynamical Model of Educational Effectiveness though Gamification', and to widely announce a pure and right function of games through our model and simulation. Previous papers have proposed a dynamical model for the Gamification of learning [1]. We analyzed the interrelation of dozens of elements from GDF(cool features, fancy graphics, challenging puzzles, and an intriguing setting and story)[2], KCLG(challenge, curiosity, fantasy and control) [3], ARCS(attention, relevance, confidence and satisfaction)[4] and MDA(mechanics, dynamics and aesthetics) [5-7] using our empirical techniques, and then extracted Four Primary factors-*Curiosity, Challenge, Fantasy and Control* - for Educational Effectiveness through the Gamification of learning. We also created a sigmoidal equation for the educational effectiveness of Gamification by analyzing and correlating these factors.

2. Dynamical Model for the Gamification

The core idea of the model for educational effectiveness through Gamification as a function of time, $x(t)$, initiates the comparison with an assumption of educational effectiveness of traditional learning. Assume that the educational effectiveness of the traditional way is solid but steady which is why it is assigned a constant value, $\boldsymbol{E_{TW}}$. The dynamical model initially starts with a lower educational effectiveness than the traditional way. Thereafter the educational effectiveness increases rapidly and exceeds the educational

effectiveness of traditional learning and converges with learning capacity. Based on the assumption above, at the initial stage, the rate of change of educational effectiveness with respect to time is proportional to $x(t)$,
$\frac{d}{dt}x(t) \approx Gx(t)$, where G is the growth rate of educational effectiveness through Gamification, and $x(t)$ will converge on a learning capacity L, $\frac{d}{dt}x(t) = 0$ at $x(t) = L$. The mathematical expression is given by

$$\frac{d}{dt}x(t) = Gx(t)\left(1 - \frac{x(t)}{L}\right). \quad (1)$$

Assume that the idea representing $x(t)$ is a sigmoidal curve so that the educational effectiveness function through Gamification can be established as

$$x(t) = \frac{a}{b+ce^{-d(t-k)}}\ , \quad (2)$$

where a, b, c, d, and k are non-negative constants. The analytical analysis and comparison with equation (1) make the constants G, L, a, b, c, d, and k valuable.

After some algebra the first derivative of the equation (2) with respect to time is

$$\dot{x}(t) = d\,x(t)\left(1 - \frac{b}{a}x(t)\right) \quad (3)$$

Comparisons can be made between equations (1) and (3) with the constant d as the growth rate of educational effectiveness through Gamification, G. Since the *Control* suggests that if the user can dominate the game, more time will be spent in the game, d can be considered as the *Control* in this model. The reciprocal of $\frac{b}{a}$, $\frac{a}{b}$, is learning capacity, L, in equation (1). Assume that the relatively high *Challenge* and relatively low *Fantasy* reduce the rate of change of educational effectiveness with respect to time respectively. So, b and a are denoted as *Challenge* and *Fantasy* respectively. Note that the inflection point, $\left(\frac{1}{d}\ln\frac{c}{b} + k,\ \frac{a}{2b}\right)$, in equation (2) needs to be positive. So that the following condition, $c > b$, is obtained. The condition, $c > b$, states that the constant, c, is relatively higher than the constant, b, to keep accelerating the educational effectiveness. Since the constant, b, is *Challenge*, the assumption of the constant, c, as *Curiosity* is understandable.

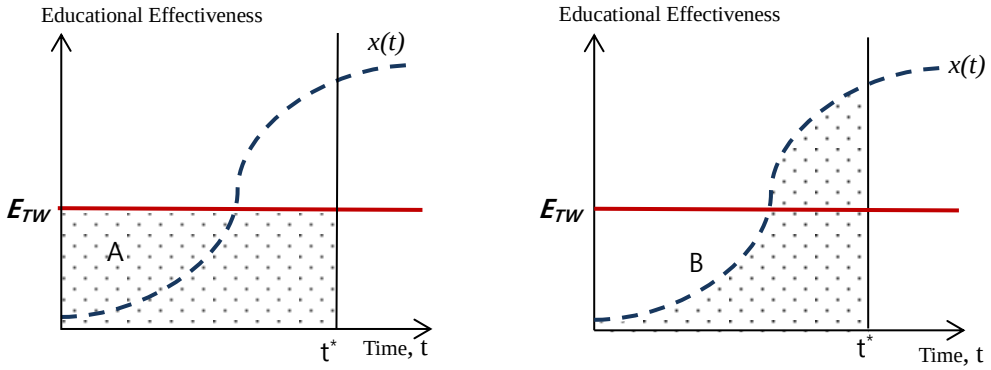


Figure 1. 'A' in the Left Figure States that the Area of E_{TW} from t=0 to t=t* and 'B' in the Right Figure States that the area of x(t) from t=0 to t=t*

To consolidate the idea of the coefficients, *a*, *b*, *c*, *d*, and *k*, the relationship between $x(t)$ and $\boldsymbol{E_{TW}}$ should be considered. In Figure 1 above, to take advantage of the educational effectiveness through the Gamification, it needs to satisfy the area of $\boldsymbol{E_{TW}}$ from *t=0* to *t=t** must be less than the area of *x(t)* from *t=0* to *t=t**. Then the result is given by

$$\boldsymbol{E_{TW}}\,\mathrm{t}^* < \frac{a}{b}\left[t^* + \frac{1}{d}\ln\left(\frac{b+ce^{-dt^*}}{b+c}\right)\right]. \tag{4}$$

2.1. The Characteristics of the Coefficients

1) *d* is the growth rate of educational effectiveness through gamification and can be considered as the *Control* in this model.

2) *b* and *a* are denoted as *Challenge* and *Fantasy* respectively and the value of $\frac{b}{a}$ is relatively low but not too low for decreasing the converge time.

3) *c* is relatively higher than the constant, *b*, to keep accelerating the educational effectiveness and the constant, c, as *Curiosity*.

4) *k=0* at this point so that *x*(0)$=\frac{a}{b+c}$.

3. Reasonable Values of Coefficients through an Example

If the amount of vocabulary words that can be learned through traditional methods of learning for 1 hour is 40 words, $\mathrm{A} = \boldsymbol{E_{TW}}\,\mathrm{t}^*$, is 40 words. Where t*=1 and $\boldsymbol{E_{TW}}$=40.Assume that the educational effectiveness through Gamification at t*=1 and *k*=0 will converge to 80% of stable fixed point, $\frac{a}{b}$, so that from equation (2),

$$ce^{-d} = \frac{b}{4}. \tag{5}$$

With the condition above, the equation (4), $\boldsymbol{E_{TW}}\,\mathrm{t}^* < \frac{a}{b}\left[t^* + \frac{1}{d}\ln\left(\frac{b+ce^{-dt^*}}{b+c}\right)\right]$, at t*=1 , gives

$$40 < \frac{a}{b}\left[1 + \frac{1}{d}\ln\left(\frac{5b}{4(b+c)}\right)\right] \tag{6}$$

Assume that the initial point, *x*(0) = $\frac{a}{b+c}$ is 20% of the stable fixed point,$\frac{a}{b}$, so that $\frac{a}{b+c} = 0.2\frac{a}{b}$ then

$$c = 4b, \tag{7}$$

Substituting equation (7) into equation (6),

$$40 < \frac{a}{b}\left(1 - \frac{1.3863}{d}\right), \tag{8}$$

so that $d > 1.3863$ and from equation (5) and equation (7) the value of *d*=2.7726. The value of *d* makes equation (8)

$$\mathrm{b} < \frac{a}{80} \tag{9}$$

The final assumption is that the initial point, $x(0)= \frac{a}{b+c}$, and using equation (7) the initial point can be defined as $x(0) = \frac{a}{5b}$. Since the result of equation (9) states that it is at least 40% of $\boldsymbol{E_{TW}}$ and is a reasonable value as the initial point. So assume 50% $\boldsymbol{E_{TW}}$ is the initial condition, $\frac{a}{5b} = 20$. Hence $a = 100b$.

3.1. Conclusion from the Example

From the example above d=2.7726, $a = 100b, c = 4b$ and $a = 25c$.The value of fantasy is much higher than the value of challenge and the value of curiosity is 4 times higher than challenge. Using the relationship of the coefficients, the inflection point, $\left(\frac{1}{d}\ln\frac{c}{b} + k, \frac{a}{2b}\right)$is (0.5, 50). The simulation of the example is presented in Figure 2 below. To find the intersection of $\boldsymbol{E_{TW}}$ and equation (2), set $40 = \frac{10}{0.1+0.4e^{-2.7726t}}$, and the intersection point given by (0.3538, 40).

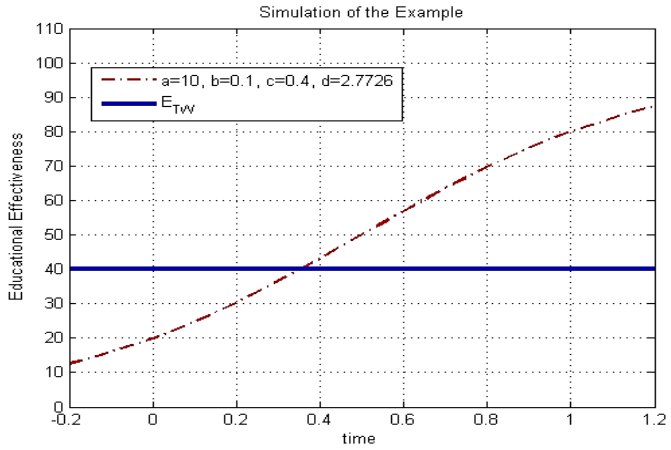


Figure2. To Satisfy the Conditions in the Example, Set a=10, b=0.1, c=0.4, and d=2.7726

All the coefficients satisfy the condition in the example above. The example states that the ratio of fantasy to challenge is 100 which is 25 times higher than the ratio of curiosity to challenge. Note that the ratio is the relationship ratio between two factors, not the priority of the factors. In other words, the ratio of fantasy to challenge being 100 does not mean that fantasy is 100 times more important than challenge. As mentioned above, the absolute meaning of the values for the coefficients can be defined through additional research. The phenomenon of the simulation shows us that at the beginning, educational effectiveness through Gamification is just 50% of the traditional way but after reaching a tipping point at t=0.3538, Gamification effectiveness actually exceeds the traditional method. With the profitable condition of the coefficient, the educational effectiveness through Gamification per unit of time can be more efficient than the traditional way of learning.

4. Sensitivity of the Coefficient to Different Ages and Genders

4.1. Sensitivity of the Coefficient

In this section the sensitivity of the coefficient will be considered. Even though the Gamification provides the same values of the four factors as absolute values, depending on the age and gender, the values of the four factors may change. Assume that the four constants, *Sa, Sb, Sc, and Sd,* are sensitivity coefficients of *Fantasy*, *Challenge*, *Curiosity*, and *Control* respectably. Those have positive values and vary depending on age and gender.

4.1.1. Sensitivity of *Fantasy, Sa*: *Fantasy* is mainly connected with intangible items like virtual item, feedback, honor, delight, love and so no. And previous research shows that males are more sensitive than females [8]. We assume that the age group in their 20s is the most sensitive age in males and adolescence (10's) is the most sensitive age in females.

Table 1. The Sensitivity of the Fantasy, $a_{ij,}$ depends on Ages and Genders. Where i=1,2 States Male and Female, j=1, 2, 3, Represents in Age of 10s, 20s, and 30s Respectively

Sa_{ij}	Sa_{i1}	Sa_{i2}	Sa_{i3}
Sa_{1j}	1	1.1	1
Sa_{2j}	1	0.9	0.8

4.1.2. Sensitivity of *Challenge, Sb*: *Challenge* is mainly related to tangible items like quest, goal, leader board, points, levels and so on. So that assumes females are more sensitive than males. The age group in their 30s is the most sensitive age for females and adolescence (10's) is the most sensitive ages in males.

Table 2. The sensitivity of the Challenge, $Sb_{ij,}$ depends on Age and Gender

Sb_{ij}	Sb_{i1}	Sb_{i2}	Sb_{i3}
Sb_{1j}	1.1	1	0.9
Sb_{2j}	1	1.1	1.2

4.1.3. Sensitivity of *Curiosity, Sc*: *Curiosity* is mainly linked with thrill, surprise, envy, comedy, appointment, progressive unlock and so on. So that assumes females are more sensitive than males. The age 20s age group is the most sensitive age group in females and adolescence (10's) is the most sensitive age group in males.

Table 3. The Sensitivity of the Curiosity, $Sc_{ij,}$ depends on Age and Gender

Sc_{ij}	Sc_{i1}	Sc_{i2}	Sc_{i3}
Sc_{1j}	1.1	1	0.9
Sc_{2j}	1.1	1.2	1

4.1.4. Sensitivity of *Control, Sd*: *Control* is directly connected to *Fantasy*, *Challenge*, and *Curiosity*. So that the value of *Control* is an arithmetic mean of the three factors.

Table 4. The Sensitivity of the Control, Sd_{ij}, depends on Age and Gender

Sd_{ij}	Sd_{i1}	Sd_{i2}	Sd_{i3}
Sd_{1j}	1.07	1.03	0.93
Sd_{2j}	1.03	1.07	1

4.2. Simulation with Sensitivity of Ages and Genders

Based on the sensitivity above section some simulations are provided below.

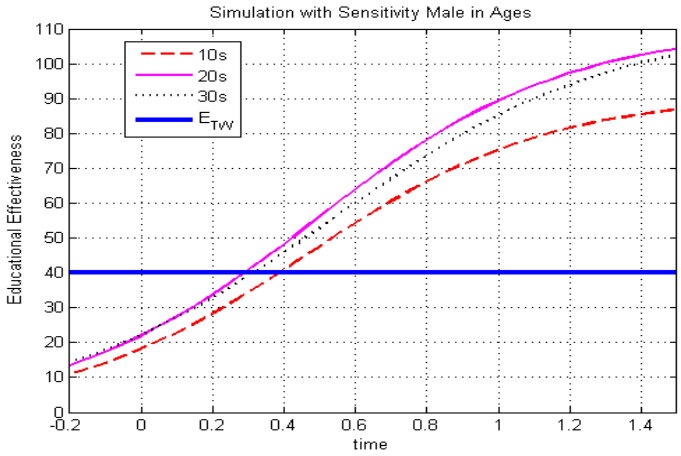


Figure 3. Using the same Coefficients in the above Example, a=10, b=0.1, c=0.4 and d=2.7726 with Sensitivity, Table 1-4. This Simulation considers Male cases in Different Age Groups, 10s, 20s, and 30s

The simulation for different ages of male is provided in the Figure 3 the males in their 10's have learned 47.3276 words, males in their 20's have learned 55.8923 words, and males in their 30's have learned 53.1816 words through Gamification in one hour. The results are 18.32%, 39.96%, and 32.95% greater than the traditional way. Also the simulation for different ages of females is provided.

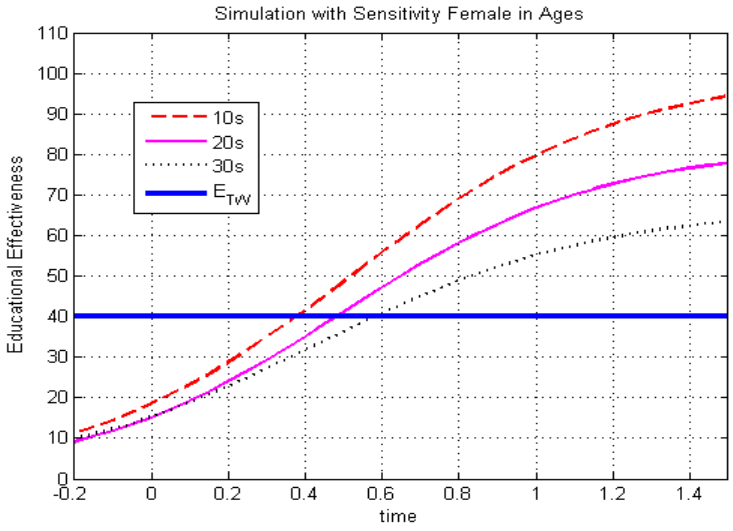


Figure 4. The same Coefficients are used in the above Example, a=10, b=0.1, c=0.4 and d=2.7726 with Different Sensitivities, Table 1~4, Different Ages for Female Cases

According to the assumption in section 4, adolescent females (10's) learned 48.8469 words, while females in their 20's learned 41.0835 words, and females in their 30's learned 35.9591 words through Gamification for one hour in the simulation, Figure 4. The results are 22.12% and 2.71% more than the traditional way but the result of the female in their 30's is 10.1% below than the traditional way. The ratios of the *Fantasy* factor to the *Challenge* factor, $\frac{a}{b}$, 0.8182 in 20s and 0.6667 in 30's from Table 1 and Table 2.

As a result the difference of the educational effectiveness through Gamification from females in their 20's to female in their 30's is 12.81% for one hour, in other words the difference between females in their 20's and females in their 30's is 5.12 words in one hour. Adolescent male and females cases (10's) range from 39.96% to 18.32% improvement in the total amount of words that can be learned through Gamification rather than the traditional way. The following simulations deliver more interesting results. Using reasonable ranges of the four primary factors based on the assumption in the examples and additional sensitivity factors applied to 100 different people in six different cases of different ages and genders. In the following six simulations, the random numbers are generated with even distribution from -30% to 30% in the sensitivity factors for 100 people. This is based on one standard deviation of the mean from the normal distribution.

4.2.1. Simulation for 100 Males in 10s with ±30% of Sensitivity

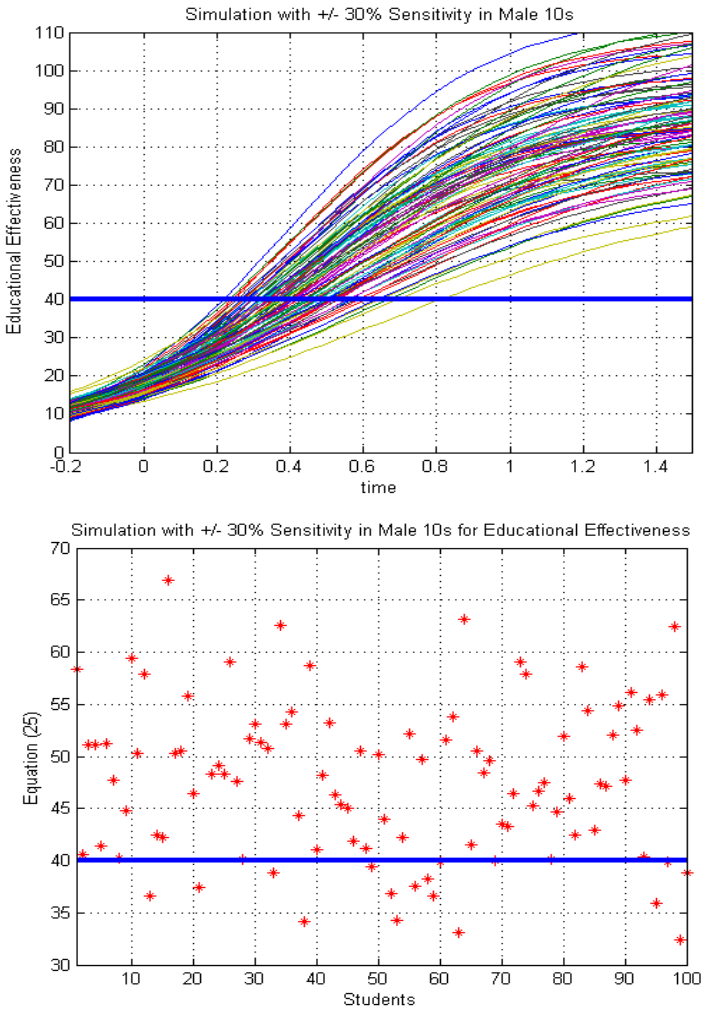


Figure 5. The Simulation for 100 Adolescent Males (10's) based on the Table 1-4 with ± 30% Sensitivity. The Educational Effectiveness of 100 Adolescent Males (10's) through Gamification over Time is Simulated on the Left Hand Side of Figure 5. The Amount of Vocabulary Words that can be learned through Gamification per 1 hour for 100 Different Individuals is represented as Asterisks in the Right Hand Side of Figure 5

In the simulation above, 83% of adolescent males have results exceeding that of traditional methods. In the result of the simulation for adolescent males, only 17 % learned less than or equal to the traditional way. Through Gamification, about 36% of the people learned 100%~120% above the traditional way and about 34% of the people learned 121%~140% over the traditional way. The average amount of learning words for 100 adolescent males is 47.3 which is 18.25% more than the traditional way.

4.2.2. Simulation for 100 Adolescent Females (10's) with ±30% Sensitivity

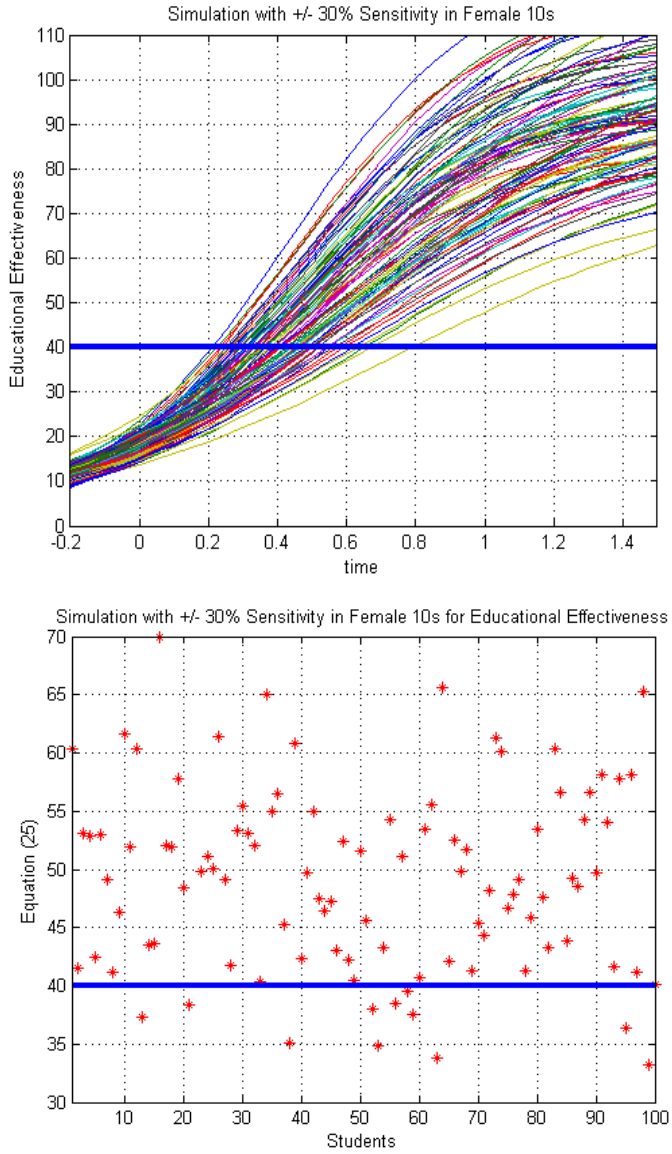
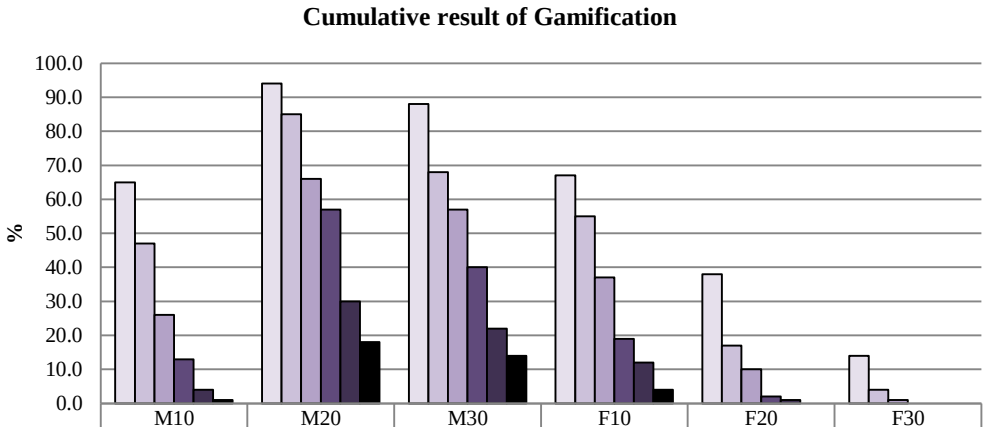


Figure 6. The Simulation for 100 Adolescent Females based on the Table 1-4 with ±30% Sensitivity. The Educational Effectiveness of 100 Adolescent Females through Gamification over Time is simulated in the Left Hand Side of Figure 6. The Amount of Vocabulary Words that can be learned through Gamification per One Hour for 100 Different Individuals is represented as Asterisks in the Right Hand Side of Figure 6

In the simulation above, 87% of adolescent females had higher results using gamification as compared to the traditional way. As a result in adolescent females, only 13 % learned less than or equal to the traditional way. Though Gamification, about 33% of the females learned 100%~120% of the traditional way and about 35% of them learned 121%~140% of the

traditional way. The average amount of learning words for 100 adolescent females is 48.9 which is 22.25% above the traditional way. Note that the result of the 100 adolescent females was about 4% better than the 100 adolescent males. The following charts are the summary of the simulations, Figure 5-Figure 6 and the other four cases.



	M10	M20	M30	F10	F20	F30
10%	65.0	94.0	88.0	67.0	38.0	14.0
20%	47.0	85.0	68.0	55.0	17.0	4.0
30%	26.0	66.0	57.0	37.0	10.0	1.0
40%	13.0	57.0	40.0	19.0	2.0	0.0
50%	4.0	30.0	22.0	12.0	1.0	0.0
60%	1.0	18.0	14.0	4.0	0.0	0.0

Figure 7. Cumulative Result of the Six Cases

7. Conclusion and Future Work

When the four primary factors have profitable values such as *d*=2.7726, $a = 100b, c = 4b$ and $a = 25c$, the educational effectiveness through Gamification can easily exceed the educational effectiveness using traditional methods, $\boldsymbol{E_{TW}}$. According to the example the result shows that at the beginning, the educational effectiveness through Gamification is just half of the traditional way but the total amount of vocabulary words that can be learned through Gamification during one hour is 25% greater than the traditional way.

The sensitivity of the coefficient is also considered. In this research, only the sensitivity of age and gender is considered for the first step. Defined are four constants; *Sa, Sb, Sc, and Sd,* for sensitivity of the four primary factors. The sensitivity will vary depending on age and gender. Even though the values of the sensitivities don't have a clear scale, a fresh method of approach is considered. The more reasonable ranges of the Sensitivity will be considered in future research. According to the simulation with sensitivities above, Figure 3-4, the result of the educational effectiveness through Gamification can be varied depending on the age and gender of the participants. Even though the different ages and genders start with similar initial points, the results vary from above 39.9% to below 10.1% of the traditional way. The interesting simulations for 100 different individuals are provided in Figure 5-6. In subsequent research, the ranges of sensitivities will have more reasonable values through experimental and statistical data.

In upcoming work, we will systemize the dynamical model through substantive verification and we will focus on reconsolidating the dynamical model for the theoretical

foundation of Gamification which can positively affect aspects of society including but not limited to Health, the Environment, and Government.

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