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Error and Erasure Decoding Scheme for RS Codes in MFSK based FH/SSMA Communication

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Abstract

In this paper, we propose an effective error and erasure decoding scheme with multi-level erasure-decision threshold for Reed-Solomon (RS) codes in frequency-hopped multiple-access communications with M-ary FSK signaling, and analyze its performance.RS coding is used to correct erasures and errors caused by other-user interference and thermal noise. The proposed decoder decides whether to erase the received RS code symbol based on the erasure decision threshold. The decision formulas for the optimal erasure decision threshold are derived in such a way that packet error probability can be minimized. Numerical results show that the proposed multi-level threshold control decoding scheme yields a higher throughput over a conventional decoding cases.

Keywords: *Error and Erasure, RS Codes, Frequency Hopped*

1. Introduction

It is well known that some form of forward error-correction is required to obtain acceptable performance in frequency-hopped multiple-access communication systems. For most error and erasure correction decoding schemes, erasures are preferable to errors because more erasures than errors can be corrected. If the communication receiver has a way to determine the reliability of the received symbols, it is advantageous to declare the erasure of the least reliable symbol prior to decoding operation.

Hence, the combination of block coding and error-erasure decoding can enhance the performance substantially in frequency-hopped multiple-access communication systems if a good method is employed to decide which symbols should be erased. This paper is concerned with the error and erasure decoding schemes for RS codes in frequency-hopped multiple-access systems with M-ary FSK signaling. RS coding is used to correct erasures and errors caused by other-user interference and thermal noise. It is known that if it is possible to obtain some reliable side information on the received symbols which have been corrupted by other users, then those unreliable symbols can often be accurately erased. There are several methods to generate the side information to identify unreliable symbols at the decoder input in a frequency-hopped multiple-access systems.

They are well described in [1-2].

Recent Publications on the performance of coded frequency-hopped spread-spectrum communications consider mainly the RS coding with errors-only decoding, erasure-only decoding and errors and erasures decoding [3-4].

In this paper, we investigate the error and erasure decoding scheme with multi-level erasure decision threshold for frequency-hopped multiple-access systems with M -ary FSK signaling.

The proposed decoding scheme carries out the adaptive erasure-decision process according to the channel traffic prior to decoding process. For performance comparison, two conventional decoding schemes are considered.

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Error-erasure decoding based on erasure insertion under perfect side information, is considered. In this system, the decision device erases each symbol that has been corrupted by other user. The other is the standard hard-decision decoding scheme with *M*-ary FSK signaling.

In Section 2, we introduce the system model considered in this paper. In Section 3, we propose an error and erasure decoding scheme with multi-level erasure-decision threshold and analyze its performance. In Section 4, the performance evaluation of proposed decoding scheme is provided for various channel traffic level. Finally, conclusions are made in Section 5.

2. System Model

We consider a Frequency-Hopped Multiple-Access (FH/MA) communication system that consists of q orthogonal narrow-band frequency slots.

We also consider a N transmitter-receiver pairs which communicate each other simultaneously over a fixed channel bandwidth. We assume that the use of non-coherent orthogonal M-ary FSK; each frequency slot consists of M orthogonal tone positions.

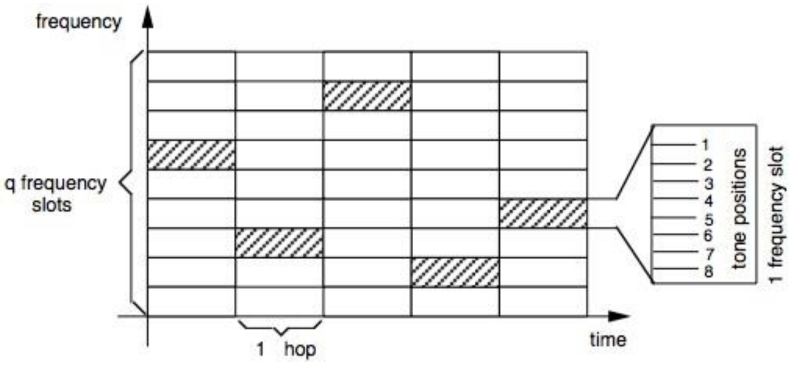

Figure 1. Frequency-Hopping Channel with One M-ary FSK Symbol Transmitted per Hop (Example Shown for *M* = 8)

Figure 1 illustrates this signaling scheme. Each transmitter-receiver pair has a unique hopping pattern that selects the frequency slot. We model the hopping patterns as independent, identically distributed (i.i.d.) random sequences uniformly distributed over a set of q frequencies. Each user of the channel transmits one fixed length string of symbols, called a packet, in each time slot. We assume that each packet consists of a Reed-Solomon (RS) codeword, and one code symbol (M-ary), which contains a single tone representing $\log_2 M$ bits, is transmitted in one hop. We also assume that a synchronous frequency hopping system, which requires that the dwell interval for the desired signal and all interfering signals are perfectly aligned at the receiver. If two or more active transmitters transmit in the same frequency slot at the same time, then a "hit" occurs. The probability that another packet has the symbol transmitted in the same frequency slot at any time during the hop duration for that symbol is called the probability of a hit, denoted by p_h, and is given by [5].

$$p_h = \frac{1}{q} \tag{1}$$

In this situation, the demodulator can determine whether a symbol of interest is hit. We call this as the case of side information available and the symbol being hit is erased. If we let K be the active number of users wishing to transmit the packet in a given hop, the probability of a symbol being erased given *K* simultaneous transmissions with perfect side information is thus given by

$$P_{er} = 1-(1-p_h)^{K-1}, \qquad 1 \le K \le N \tag{2}$$

In case of no hit, if we have assumed a slow Rayleigh fading channel, the probability of symbol error, P_e, will be given by

$$P_e = (1-p_h)^{K-1} P_0(M) \tag{3}$$

Where the conditional probability of an *M*-ary symbol error given no hit, $P_0(M)$ in (4), on a

Rayleigh fading channel is given by [6]

$$P_0(M) = \sum_{i=1}^{M-1} \binom{M-1}{i} \frac{(-1)^{i+1}}{i+1} \cdot \frac{1}{1+\frac{i}{i+1}(\log_2 M)\frac{E_b}{N_0}} \tag{4}$$

Where $\frac{E_b}{N_0}$ is the average received signal-to-noise ratio at the receiver.

Based on (2) and (3), we can evaluate the decoder performance of conventional error-erasure decoding with perfect side information. Since (*n*, *k*) RS code can correct up to *s* erasures and *t* errors provided that $2t+s \le n-k$ [7], the packet error probability, P_E , given *K* - 1 other users, is given by

$$P_E = \sum_{\substack{s+t\le n \\ 2t+s>n-k}} \binom{n}{s,t} P_{er}^s P_e^t (1-P_{er}-P_e)^{n-s-t} \tag{5}$$

When the perfect side information is available.

The normalized throughput, *W*, defined as the average number of successfully transmitted (or correctly received) information bits per sec per Hz given *K* simultaneous transmissions, is given by

$$W = \frac{k \log_2 M}{n\,M\,q} \cdot K \cdot (1-P_E) \qquad \text{(bits/sec/Hz)} \tag{6}$$

3. Proposed Error and Erasure Decoding Scheme

In error-erasure decoding of RS code, since an (*n*, *k*) RS code can correct up to any combination of *s* erasures and *t* errors provided that $2t+s \le n-k$ [7], the erasure-decision criterion must be carefully selected to improve the error-erasure decoder performance. As the number of user in FH/MA systems gradually increases, the multiple-access interference also increases. Hence, for some cases of high traffic, the number of erasures may increase beyond the erasure correction capability of RS code. This results in the sudden decrease of throughput of FH/MA systems since the decoder may fail to decode the received packet correctly. Hence, it is required that the decoder should possess an effective decoding scheme to generate erasure reliably and to control the number of erasures.

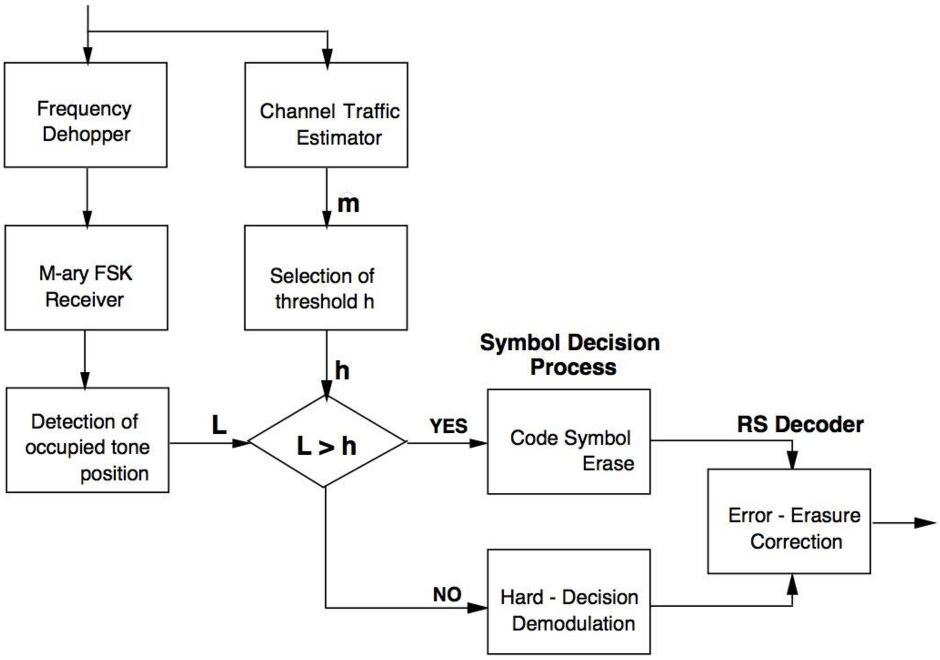

Figure 2. Proposed Error and Erasure Decoding Scheme with Multi-level Erasure-decision Threshold

The demodulator and decoder block diagram for proposed decoding scheme is shown in Figure 2. The decoding procedure is as follows. The received signal is first dehopped and passes through the non-coherent MFSK demodulator which consists of a bank of M parallel non-coherent envelope detectors, and also used in the channel traffic estimation. Then, the decoder detects the tone positions at which hits occur by other users and counts the number of tones used in the corresponding frequency slot.

This may be obtained from matched filters or envelope detector outputs of MFSK receiver. At the same time, the decoder determines the erasure-decision threshold h based on the estimated channel traffic K_{est}.

Estimated channel traffic K_{est} indicates the estimated value of active number of users in a given hop. It is not difficult to estimate the channel traffic in a frequency-hopped multiple-access systems. Perhaps the simplest way to estimate channel traffic for a frequency-hop network is the number of frequency slots that have signal strengths above a certain threshold. An alternative is to monitor a few frequency slots and count the number of times signals are present in these slots. The practical implementations of these traffic estimators are well described in [8]. In the remainder of this paper, we considered only the perfect traffic estimation scenario. The decision to erase or not is made independently for each M-ary symbol. If we denote L as the total number of distinct occupied tone positions in particular frequency slot during the transmission of one M-ary symbol, the decisions on whether to erase are then made by comparing the value of L to the predetermined threshold h. The determination procedure for the erasure-decision threshold h will be discussed later. If the number of occupied tone positions exceeds the predetermined threshold h, which is the arbitrary value between 1 and M, the corresponding code symbol is erased, and this erased symbol will be fed to the RS decoder. If the number of the occupied tone positions does not exceed h, the decoder performs the hard-decision demodulation, *i. e.*, the decoder chooses the tone with the maximum energy as a decision symbol, and this results in either a correct or error symbol in RS decoding. After the erasure-decision process of the code symbol is completed, each symbol in an RS codeword is in one of the following three states: correct, erased, or in error. Finally, RS decoder performs the error-erasure correction that corrects the combination of both erasures and errors. The performance analysis of the proposed error-erasure decoding scheme is as follows.

The probability P_{er}, that RS code symbol becomes an erasure after symbol decision given K-1 other users, is expressed as

$$P_{er} = \sum_{m=0}^{K-1} P(er \mid m) \cdot Q(m \mid K) \quad (7)$$

Where

$P(er \mid m)$ = *Pr* (symbol erasure | *m* other users in the same frequency bin as desired signal)

$Q(m \mid K)$ = *Pr* (other users in the same frequency bin as desired signal | *K* - 1 other Users in the channel).

In case the number of occupied tone positions, *L*, out of *M* tones is greater than *h*, the corresponding symbol is erased. Therefore, the conditional erasure probability $P(er \mid m)$ is given by

$$P(er \mid m) = \sum_{j=h+1}^{M} P(L = j \mid m) \quad (8)$$

Where $P(L = j \mid m)$ denotes the probability that *j* tones being occupied given *m* hits and is given by [4]

$$P(L = j \mid m) = (\frac{1}{M})^{m+1} \binom{M}{j} \cdot j \sum_{r=0}^{j-1} (j-r)^m (-1)^r \binom{j-1}{r} \quad (9)$$

Since the users are assumed to choose frequency bins independently and with uniform distribution, $Q(m \mid K)$ is obtained as

$$Q(m \mid K) = \binom{K-1}{m} {p_h}^m (1-p_h)^{K-1-m} \quad (10)$$

The probability P_e that the corresponding RS code symbol is in error given K-1 other users, is obtained as

$$P_e = \sum_{m=0}^{K-1} P(e \mid m) \cdot Q(m \mid K) + (1-p_h)^{K-1} P_0(M) \quad (11)$$

Where

$P(e \mid m)$ = *Pr* (symbol error | *m* other users in the same frequency bin as desired signal)

In case the number of occupied tone positions, *L*, is less than *h*, the symbol decision process carries out the hard-decision demodulation that selects one tone with maximum energy out of *M* tones.

Therefore, the conditional error probability $P(e \mid m)$ is given by

$$P(e \mid m) = \sum_{j=1}^{h} (\frac{j-1}{j}) \cdot P(L = j \mid m) \quad (12)$$

The proposed error and erasure decoding scheme with $h = M$ represents the conventional hard-decision decoding scheme.

The packet error probability P_E and normalized throughput W can be obtained through (5) and (6) with P_{er} and P_e in (7) and (11), respectively.

In addition, an important parameter we must consider is the threshold *T*, which determines whether the RS code symbol should be erased or not.

Intuitively, when the channel traffic is low, the erasure-decision threshold *h* have the small value since the number of erasures is relatively small. However, as the channel traffic gradually increases, the threshold *h* should be increased for adjusting the ratio of erasures to errors to optimal level in order to maximize the throughput of FH/SSMA systems.

Consequently, the determination of the erasure decision threshold *h* will be some form of several level threshold change based on the channel traffic estimation.

For example, if we let $\overline{m}$ as the estimated channel traffic value, *M* - level threshold control scheme means that $h = 1$ is selected for $\overline{m} \in \{1, 2, \cdots, m_1\}$, $h = 2$ is selected for $\overline{m} \in \{m_1 + 1, m_1 + 2, \cdots, m_2\}$, $\cdots$, $h = M$ is selected for $\overline{m} \in \{m_{M-1} + 1, m_{M-1} + 2, \cdots, K\}$ for some $m_1, m_2, \cdots, m_{M-1}$. The values of $m_1, m_2, \cdots, m_{M-1}$ can be determined experimentally for given *M*, *q* and *K*.

4. Numerical Results

In this section, we present some numerical results about the performance of proposed error and erasure decoding scheme described in the previous Section. RS-(7,3) codes are used and its coding rate is approximately equal to 1/2. The performance measure we have considered is the normalized throughput defined in Section 2.

Figure 3 and Figure 4 represent the normalized throughputs *W* versus the number of active users in the channel in variation to the erasure decision threshold *h*. These figures have the parameter values of *q* = 50 frequency slots, *M* = 8 and E_b / N_0 = 12 dB and 20 dB, respectively.

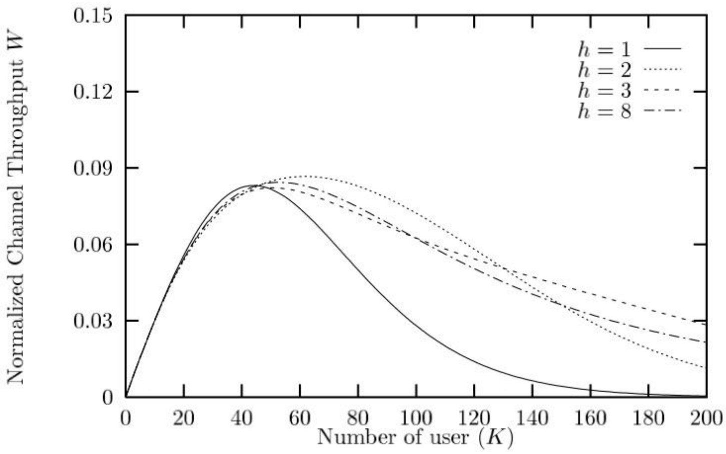

Figure 3. Normalized Throughput versus the Number of Active Users for Various Erasure-Decision threshold *h* ; RS-(7,3) Code, *M* = 8, *q* = 50, E_b / N_0 = 10 dB

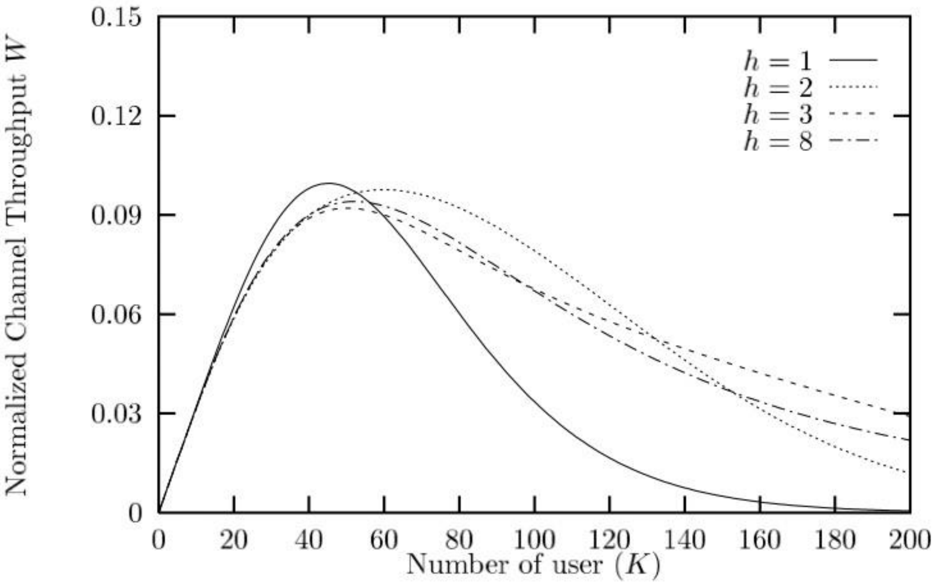

Figure 4. Normalized throughput Versus the Number of Active Users for Various Speed Erasure-Decision threshold *h*; RS-(7,3) Code, *M* = 8, *q* = 50, E_b / N_0 = 20 dB

From these Figures, it can be observed that for low traffic, the highest throughput is attained when h = 1. But as the channel traffic gradually increases, the value of h for the highest throughput is changed to 2. In the high traffic case, h = 3 achieves the highest throughput. When h is greater than 3, little difference in the throughput is observed with the variance of h. Hence, only h = 8 case is shown in those figures. Specially, h = 8 case (*i. e.*, h is equal to M) corresponds to the hard-decision decoding case.

The reason for this is as follows. When the traffic is low, the possible number of hits by other users is relatively small. Hence, it is sufficient to correct all the erasures produced by other users' hit with the given erasure correction capability. But as the channel traffic gradually increases, it needs the control algorithm for adjusting the number of erasures to prevent the decoding failure. These figures clearly indicate that the multi-level threshold control decoding scheme can attain the highest throughput among the possible values of erasure-decision threshold, provided that we control the threshold h accurately according to channel traffic.

Figure 5 and Figure 6 compare the normalized throughputs W of 3-level threshold control scheme with those of conventional decoding schemes. Hard-decision decoding scheme (h = M case in proposed decoding scheme) and error-erasure decoding scheme with Perfect Side Information (PSI) are considered for performance comparison.

It has been shown that the proposed 3-level threshold control decoding scheme achieves a higher throughput than conventional decoding scheme in the all range of channel traffic. As E_b / N_0 gradually increases, it is observed that the throughput difference between the multi-level threshold control and conventional decoding scheme grows a bit larger.

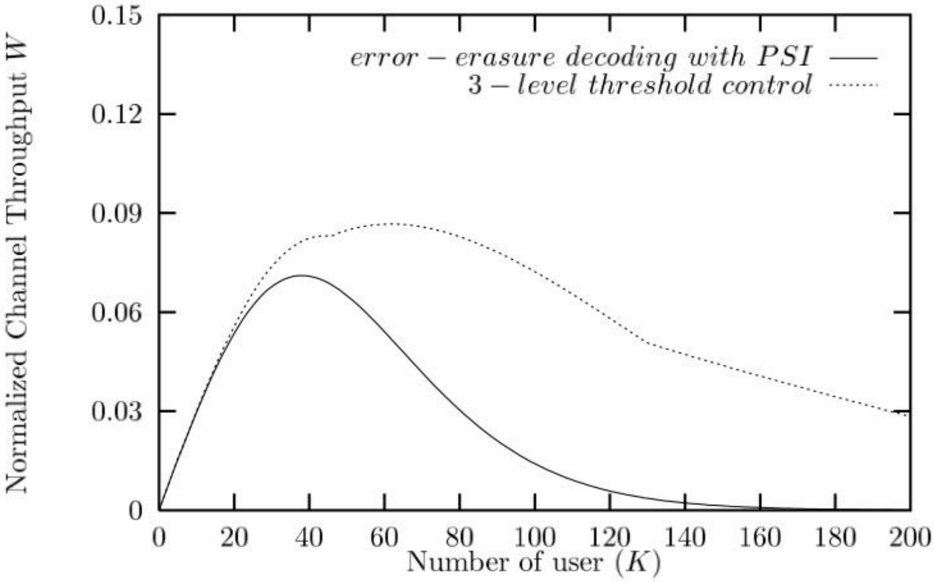

Figure 5. Comparison of Normalized throughput versus the Number of Active Users; RS-(7,3) Code, *M* = 8, *q* = 50, E_b / N_0 = 10 dB

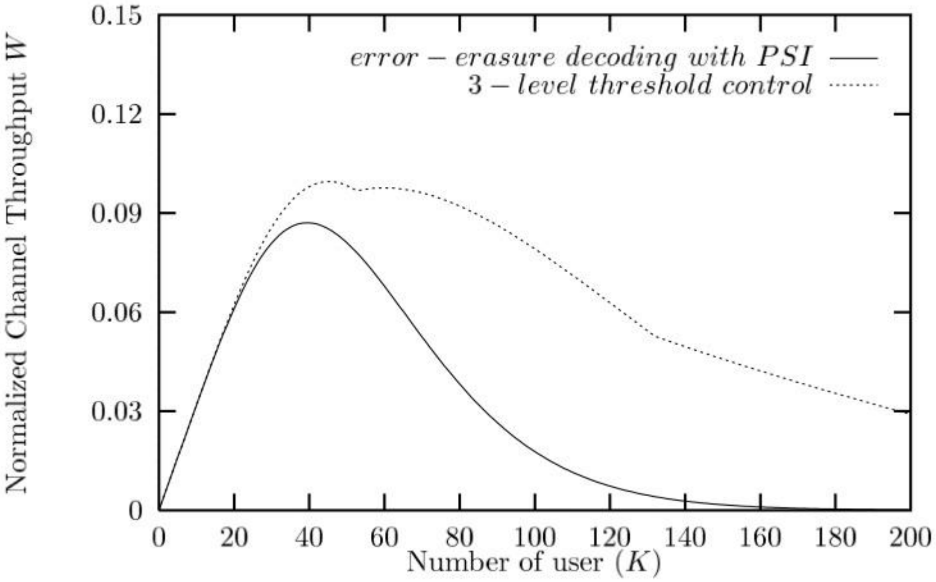

Figure 6. Comparison of Normalized throughput versus the Number of Active Users; RS-(7,3) Code, *M* = 8, *q* = 50, E_b / N_0 = 20 dB

The reason for this is that when E_b / N_0 is low, the transmitted symbols are dominantly affected by thermal noise as well as multiple-access interference, this results in many errors as well as erasures, hence the control effect of the number of erasures doesn't appear well.

But, as E_b / N_0 increases, since the multiple-access interference becomes more dominant than thermal noise, the threshold control becomes more effective. These figures clearly indicate that the proposed multi-level threshold control decoding scheme is very effective to increase the normalized throughput for frequency-hopped multiple-access systems with RS codes and *M*-ary FSK signaling.

5. Conclusion

The error and erasure decoding schemes for RS codes in frequency-hopped multiple-access communication system employing *M*-ary FSK signaling have been studied. The multi-level threshold control decoding scheme, in which the decoder can adaptively change the erasure-decision threshold according to channel traffic, has been proposed, and its performance has been analyzed. Numerical results have shown that the multi-level threshold control decoding scheme yields a higher throughput than conventional decoding cases do.

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